Measuring the Chaos of Water Droplets Using Photo-diode Sensors

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In our study, we investigated water droplet dynamics by varying the flow rate. Using two photo gates, we observed period bifurcation and chaos in the time intervals between successive droplets. Changes in the flow rate led to transitions in temporal patterns, illustrating the periodic and chaotic behavior inherent in droplet flow.

I. INTRODUCTION

The aim of our experiment is to illustrate period-doubling behavior and the transition into the chaotic regime through varying the flowrate of water droplets from a nozzle. Interestingly, we observed a situation where it appeared that pure period doubling did not occur; instead, some periods collapsed into one another, reminiscent of chaotic systems like Van der Pol oscillations [2]. Despite the challenge of adjusting the bifurcation parameter, we achieved a relatively good representation. We calculated the Feigenbaum constant as 5.9 for our water droplet system.

II. THEORY

Chaos refers to a state of apparent randomness and unpredictability in a system. Studying chaos reveals the inherent complexity of various systems, where small changes to initial conditions can lead to significantly different outcomes. In this experiment, we investigate the chaos arising from a common household occurrence – a leaky faucet. The repetitive drip of water from a leaky faucet may seem routine, but it conceals a fascinating aspect of nonlinear dynamics and chaos. Recognized as a chaotic system in 1982 [2], the experiment provides an avenue to explore the chaotic patterns emerging from water droplets falling. By observing the time intervals between water droplets falling as we tighten or loosen our ‘faucet’, we observed a period-doubling effects where periods may become very short as droplets fall quickly in succession, or longer as the pressure in the faucet is relieved. As the flow of water increases, this rhythmic behavior disappears, descending into the chaotic regime.

III. APPARATUS

Our experimental setup starts with a water container that serves as a source of water. Connected to this container by tubing is a nozzle with a control knob, allowing regulation of water droplet flow, which is then connected to a pipette tip with a 1.5 millimeter diameter. Positioned vertically below the tip is a beaker, ready to capture the falling water droplets. To quantify the flow rate, a precision scale is connected to the computer to monitor the increase in the volume of water in the beaker over time. The program LoggerPro records the change in volume over a 180 second interval.

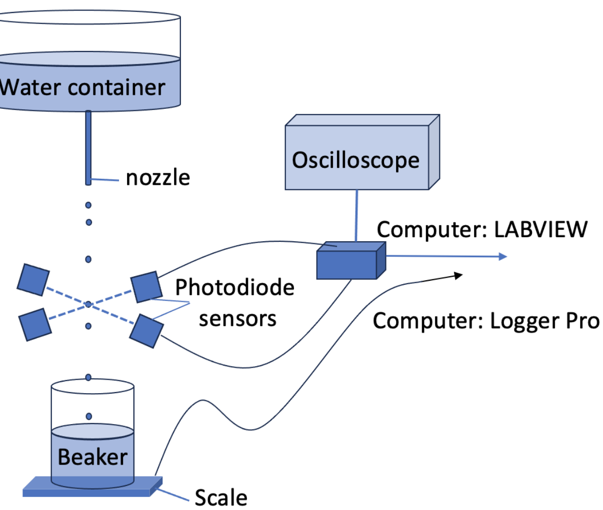

FIG. 1. Experimental setup for observing periodicity and chaos of water droplets with photogates

Two Vernier VPG-BTD photogates are positioned to capture the presence of a water droplet in time, allowing the monitoring of period doubling and chaos. Placing the photogates at different positions intersecting at the same spot on the path of the water droplet provides redundancy, ensuring that if one of the sensors misses the capture of a water droplet, the second sensor will detect it. These photogates convert light into electrical current. The photogate uses an infrared beam, and when an object passes through the gate, it interrupts this beam. An unblocked state is represented by an output signal of 5 volts, and a blocked state is represented by a signal of 0 volts. Voltage values between 0 and 5 represent partially blocked states. An oscilloscope is connected to the photogates to monitor the redundant pulses of water droplets sensed and allow proper alignment prior to the experiment. The voltage values are collected by a LABVIEW program.

IV. EXPERIMENTAL PROCEDURE

We changed the bifurcation parameter, the flow rate that causes period doubling and chaos, by controlling the flowrate with the knob on the nozzle. We assessed flow rates by monitoring the increase in water volume within the beaker over a 180-second interval. Using the LoggerPro program, we recorded data at 1-second intervals, capturing the increasing weight of the beaker over time. To quantify the flow rate, we applied linear regression analysis to the collected data, fitting a straight line to reveal the rate of volume increase per unit time.

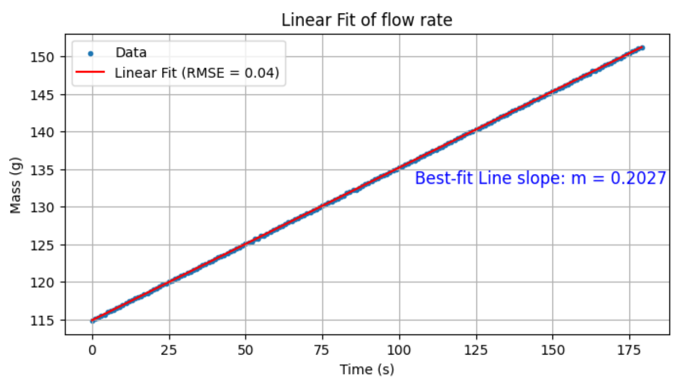

FIG. 2. A graph of the increase in the mass of water in the beaker over a 180 second interval. The slope reveals the rate of flow of the water droplets.

Figure 2 reveals a flow rate of 0.203 grams per second, or 12.16 grams per minute, which is the parameter for our 5th sample plot, refered to as μ. We organize our Poincare plots by flowrates in grams per minute.

We programmed the LABVIEW software to collect samples of the photogate signals at 10,000 Hz for 120 seconds, giving us 1,200,000 data points. We uploaded the samples to Google Collab, analyzing it using Python Pandas and Numpy libraries. We took the logical OR operation on the signals from both photogates to determine points of detection, identifying the moments when the droplet blocks at least one of the beams. Even if one gate is misaligned, the OR logic can capture the event. The time intervals between droplets were subsequently calculated based on the number of sequential samples that suggest a blocked state of 0 volts. As our flow-rate increased, the number of distinct periods increased until chaos was reached.

V. ANALYSIS

After data was collected for different bifurcation flowrate values, the following period graphs were obtained:

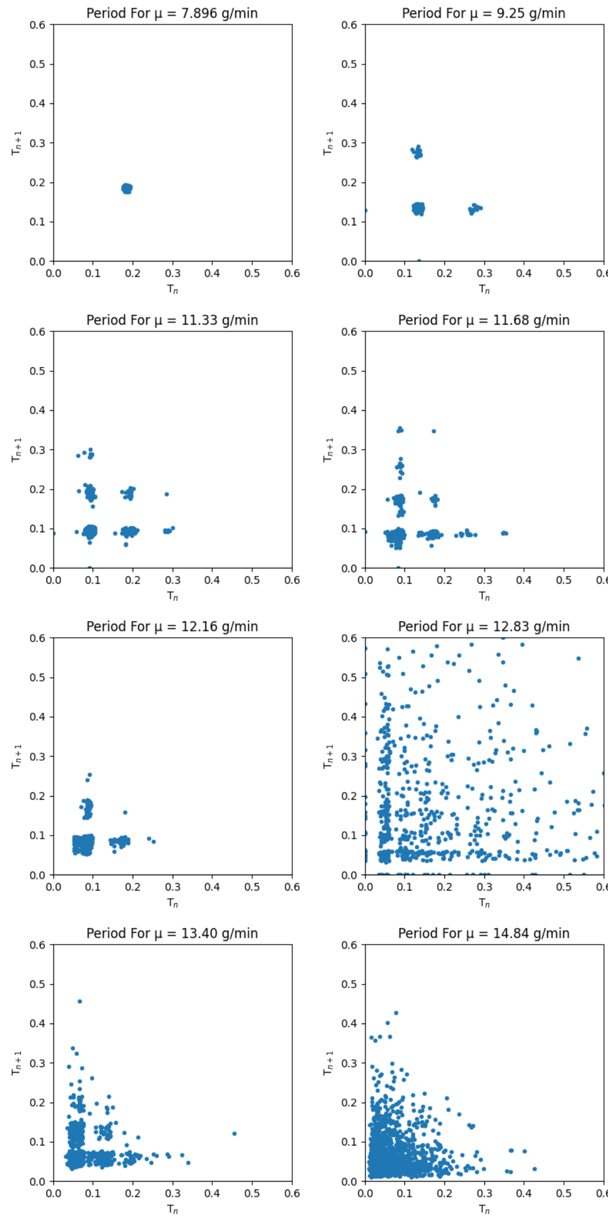

Fig 2. This figure shows how the bifurcation value changed the number of periods that our droplets had.

The inherent period doubling observed in systems transitioning into chaos is evident in our findings [3]. The apparent reduction in period from 11.68 g/m to 12.16 g/m, though less common, occurs in certain dynamic systems [1]. This observed period doubling is indicative of the onset of chaotic behavior. The Feigenbaum constant is calculated as:

$$\delta = \lim_{x\to\infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} \tag{1}$$

$$\approx \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} \tag{2}$$

$$\approx 5.983 \tag{3}$$

This value is a 21 % deviation from the expected theoretical value of 4.669. The variance from the expected value is attributed to the challenging nature of precisely determining the bifurcation parameter in our experimental setup, as well as our systematic errors detailed in the following section.

A. Systematic errors

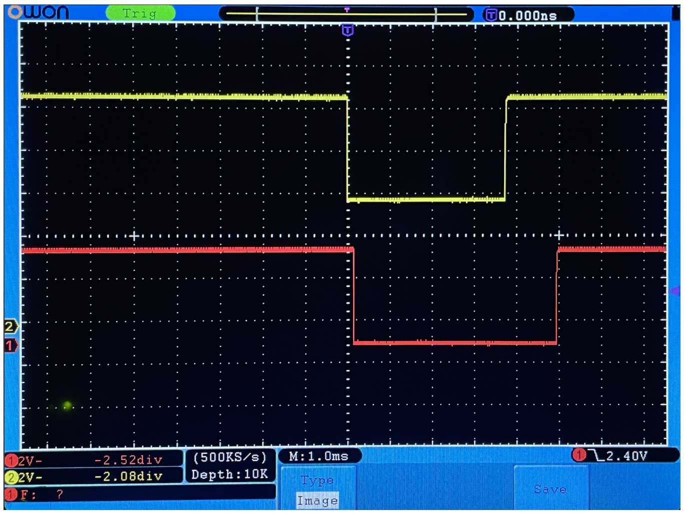

FIG. 3. The oscilloscope displays square pulses corresponding to signals from the two photogates, indicating the detection of water droplets. These pulses have a duration of around 10 milliseconds. The variations in pulse length are due to the alignment of the photogates.

We tried to eliminate systematic errors from the alignment of the photogates by adjusting the sensors based on their pulses on the oscilloscope. However, we still noticed varying signal pulse lengths and staggered pulses from the oscilloscope, such as in Figure 3. This misalignment can lead to discrepancies in the timing of events and, consequently, errors in measured time intervals. We analyzed the cross correlation of the signals to determine the best shift for the samples collected to align the signals. We found that across 13 trials, an average of 9 sample shifts would best align our data, which translates to an average misalignment of 0.9 ms. This means that on average, since we applied a logical OR to calculate our time intervals, they are calculated 1.8 ms shorter than the actual time. This effect translates to approximately a 2% deviation for calculated periods of .1 s and an approximate .6% deviation from a periods of .3 s.

VI. CONCLUSION

The period doubling continued to increase until reaching the chaotic regime as the parameter, the flow rate of water droplets, was systematically increased. Although the observed Feigenbaum constants did not precisely match expectations, their proximity indicates the chaotic nature of the system.

VII. ACKNOWLEDGEMENTS

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[1] Cartwright, M. L. Balthazar van der pol. *Journal of the London Mathematical Society s1-35*, 3 (1960), 367–376.
[2] Martien, P., Pope, S., Scott, P., and Shaw, R. The chaotic behavior of the leaky faucet. *Physics Letters A 110*, 7 (1985), 399–404.
[3] Taylor, J. R. *Classical Mechanics*, 1st ed. University Science Books, Sausalito, CA, 2005.